

8 Yi hgWX`UbX`ZY]Yfh`[fUbX]cgYb 5 i ZhU_hg]Y[`VY]`XYf`DfU[iY`6UgYVU`` K YY_°

8]Y'XYi hgWXY'<YffYb!6UgYVU``!BUh]cbU`a UbbgWXUZh'VYg]Y[h 2 ghYffY]WX''%%.%'VY]a '5i ZhU_h'XYf'DfU[i Y'6UgYVU``'K YY_ &\$&) ']b'DfU["



Dfu[ž'HgW YW]Yb'!'8]Y'XYi hgW Y'< YffYb!BUh]cbU`a UbbgW UZh \Uh'Ua '&+"'\delta b]'&\\$\) ini a '5i ZhU_h'XYf'DfU[i Y'6UgYVU``K YY_Y]bYb'VYY]bXfi W_YbXYb'GhUfh'\]b[Y`Y[h'i bX'X]Y'5i gk U\`^2 ghYffY]W\g'a]h'\%\%'VYg]Y[h''8Ua]h'a Y`XYh'g]W\XXUg'HYUa ni j Yfg]W\h`]W\'ni f\@W_ž'k \A\fYbX'Yg'g]W\'Ui Z'Y]bY'FY]\Y\YfUi gZcfXYfbXYf'Gd]Y`Y'j cfVYfY]hYh''8Ug'A UhW\X'k i fXY'VYfY]hg]a 'UW\hYb'\delta b]b['j cfnY]h]['VYYbXYhž'bUW\XYa '8Yi hgW\`UbX'XYbY`ZhYb'Fi b'Yfn]Y`hY'i bX'X]Y'DUfh]Y'_`Uf'Xca]b]YfhY'''

 \pm 'XYb 'YfghYb 'VY]XYb ' \pm b]b[g''Y[hY'X]Y'XYi hgW\Y'A UbbgW\UZh'j cf i bX''VfUW\hY''VYfY]hg'gYW\g'Fi bg'Ui Z'XUg'GWcfYVcUfX"'^2 ghYffY]W\

 $\label{thm:converse} $$ \hright{ \hri$

9]b 'ghUf_Yg'HYUa

8]Y XYi hgWX Y BUh]cbU`a UbbgWX UZhž VYghY\YbX 'Ui g'&\$
hU`Ybh]YfhYb 'Gd]Y`Yfbž'k]fX 'k À\fYbX 'XYg'Hi fb]Yfg'[Y[Yb'HYUa g
Ui g'2 ghYffY]WX Ž #\U`]Yb'i bX 'XYa '; Ugh[YVYf`UbX'HgWXYWX]Yb
UbhfYhYb "'BUWX'Y]bYa ']bhYbg]j Yb'K cf`X'6UgYVU``'7`Ugg]WE i U`]Z]Yf
]a 'A Àfn'`]Y[h'XYf': c_i g'\]Yf'Ui Z'XYf'_ca a YbXYb
9i fcdUa Y]ghYfgWXUZh"8Yf'8Yi hgWXY'6UgYVU``'i bX'GcZhVU``
JYfVUbX'f86J Ł'\Uh'XYb'?UXYf'gcf[ZÀ`h]['ni gUa a Yb[YghY``hž'a]h
Y]bYa '6`]W_'Ui Z'X]Y'@Y]ghi b[Yb'XYf'Gd]Y`Yf']b'XYf'8Yi hgWXYb
6UgYVU``'@[U'f86@Ł"'

6YgcbXYfg`\Yfj cfni \YVYb`g]bX`XfY]`G\cch]b[`GhUfg`XYf'; Ui h]b[

♣DX]Ubg. '9`]Ug`<i VYfž'HcV]Ug`A Ui ``i bX`D\]`]dd`? `YY\Ui dh"

8 UbYVYb`g]bX`Ui W\`YfZU\fYbY`Fci h]b]Yfg`k]Y`9f]W6fYb_`i bX

A Uf_i g`Gc`VUW\`j cb`XYb`6cbb`7Ud]hU`g`gck]Y`5`YI UbXYf`GW\a]Xh
j cb`XYb`; i [[YbVYf[Yf`@Y[]cbÀfYb`FY[YbgVi f['j YfhfYhYb"`8Yf
[YgUa hY`?UXYf`gYhnh`g]W\`Zc`[YbXYfa U»Yb`ni gUa a Yb.

BUaY	JYfY]b

J]bWbh'5\fYbg	7c`c[bY'7UfX]bU`g
G]a cb 6Ài a Yf	; i [[YbVYf[Yf @Y[]cbÀfY
	FY[YbgVif[
9f]W6fYb_	6cbb [·] 7Ud]hU`g
D\]`]dd '? `YY\Ui dh	; Ui h]b[ˈ=bX]Ubg
HcV]UgʻA Uiʻ	; Ui h]b[ˈ=bX]Ubg
A Uf_i g'Gc`VUW	6cbb [·] 7Ud]hU`g

5 i Z']b X]Y b A W ghYb Gd]Y Y

8 UgʻbÀWX ghYʻGd]Y``ghY\h`VYfY]hgʻUaʻ`GUa ghU[ʻi a'%(.'\$`I\f
[Y[Ybʻ\=hU`]YbʻUb"`8 Yi hgWX`UbX`k]fX'j Yfgi WXYbž'XYbʻGWXki b[ʻUi g
XYaʻ5i ZhU_hg]Y[ʻa]hʻ]bʻXUgʻ_ca a YbXYʻA UhWXʻni ʻbY\a Yb"`8]Y
A UbbgWX UZhʻ]ghʻcdh]a]gh]gWXʻi bXʻ[i hʻj cfVYfY]hYhžʻi a']bʻDfU[ʻX]Y
bÀWX ghYbʻGWX f]hhYʻUi ZʻXYaʻK Y[ʻni fʻ9i fcdUa Y]ghYfgWX UZhʻni
[Y\Yb"

8 Y hU]`g	
Cfh	DfU[ž'HgWXYWX]Yb
EiY``Yb	k k k "VUgYVU``!gcZhVU``"XY
	k k k "VUgYVU``"XY
	VUgYVU``%\$\$"XY

6Ygi WXYb G]Y i bg Ui Z b!U["bYh