

D]ghcf]i g`]b`?]Yk.`8Yi hgWX`UbX ghÀf_hl _fU]bY`a]h`a]``]cbYbgWXk YfYf @i Zh\]`ZY°

Figg`UbX'j YfghÀf_h'5b[f]ZZY'UiZ'X]Y'I_fU]bYž'k À\fYbX JYfhY]X][ib[ga]b]ghYf'D]ghcf]ig']b'?]Yk'ØVYf'A]`]hÀf\]`ZY gdf]W\h"



?]Yk ž'I _fU]bY'!'Fi gg`UbX'\Uh'gY]bY'@ ZhUb[f]ZZY'Ui Z'GhÀXhY'i bX a]`]hÀf]gW\Y'N]Y`Y']b'XYf'I _fU]bY'j YfghÀf_h"'\#b'XYb'`YhnhYb'HU[Yb _Ua 'Yg'ni 'Y]bYf'a Ugg]j Yb'C ZZYbg]j Yž'VY]'XYf'X]Y'I _fU]bYf'a]h'XYf gW\`]a a ghYb'5b[f]ZZgk Y``Y'gY]h'?f]Y[gVY[]bbž'X]Y'j cf'a Y\f'U`g XfY]'*J\fYb'VY[Ubbž'_cbZfcbh]Yfh'k i fXYb"'\#b'XYf'BUW\h'Ui Z GcbbhU['j YfnY]W\bYhY'X]Y'I _fU]bY'ZUgh'' \$\$'8fc\bYbUb[f]ZZY'gck]Y'fi bX'+\$'FU_YhYb!'i bX'A UfgW\Zi[_OfdYfUb[f]ZZYž'X]Y]bg[YgUa h'' +\$'@ ZhUb[f]ZZY'Ui ga UW\hYb"'8]Yg'VYf]W\hYh'6BB"

 $I _fU]b]gWXYf^DfAg]XYbh^K c^cXma mf^GY^Ybg_m^g]Y\h^X]YgY \\ 5[[fYgg]cb^b]WAh^U^g^FYU_h]cb^Ui Z^i _fU]b]gWXY^9fZc^[Yž^gcbXYfb]$

U`g`6Yk Y]g`ZØf`Fi gg`UbXg`8Yg]bhYfYggY`Ub`: f]YXYb"`±b`X]YgYf
_f]h]gW\Yb`@U[Y`hfUZ`XYf`XYi hgW\Y`JYfhY]X][i b[ga]b]ghYf`6cf]g
D]ghcf]i g`i bUb[Y_ØbX][h`]b`?]Yk `Y]bž'i a `g]W\`ØVYf`X]Y
a]`]hÀf]gW\Y`G]hi Uh]cb`ni `]bZcfa]YfYb`i bX`ØVYf`X]Y`[Yd`UbhY
A]`]hÀf\]`ZY`ni `X]g_i h]YfYb"`8Yi hgW\`UbX`i bX`; fc»Vf]hUbb]Yb
ØVYfbY\aYb`Y]bY`: Ø\fi b[gfc``Y`VY]`XYf`A]`]hÀf\]`ZY`ZØf`X]Y
I_fU]bY`i bX`i bhYfghØhnYb`X]Y`i_fU]b]gW\Y`@ ZhUVkY\f`a]h`ZØbZ
A]``]UfXYb`9i fc`ZØf`j YfgW\]YXYbY`a]`]hÀf]gW\Y`6YXØfZb]ggYž
XUfi bhYf`A i b]h]cb`i bX`±bghUbXgYhni b["

<i a Ub]hÀfY ?? f]gY i bX n]j]`Y J Yf`i ghY</pre>

 $8]Y @ ZhUb[f]ZZY ``UVYb `hfU[]gWXY : c`[Yb `ZØf`X]Y `N]j]`VYj `O`_Yfi b["A]bXYghYbg`nk `O`Z`A YbgWXYbž`XUfi bhYf?]bXYfž`g]bX`]b j YfgWX]YXYbYb `FY[]cbYb `XYf`I _fU]bY`[YghcfVYbž`8 i hnYbXY k Y]hYfY k i fXYb j Yf`Yhnh" <math>\Rightarrow$?]Yk k i fXYb a]bXYghYbg`j]Yf`HchY [Ya Y`XYhž`YVYbgc']b `XYf`FY[]cb `7\a Y`bmn_]^" `FYhhi b[g_fÀZhYg]bX`]b `ØVYf`' \$`GhÀXhYb`i bX`8 OfZYfb`]a `9]bgUhn" `6YgcbXYfg hfU[]gWX`g]bX`X]Y`6Yf]WXhY`Ui g`XYf`FY[]cb`GWXmhca mfž`k c`nk Y]?]bXYf`i bX`Y]b`\$ [YbX`]WXYf`Xi fWX`X]Y`5b[f]ZZY`[YhOhYh`k i fXYb"` 8]Y`DfYggY`\YVh`\Yfj cfž`XUgg`X]Y`5b[f]ZZY`U`g`HYffcfU_hYb VYnY]WXbYh`k YfXYb"

8]Y`]bhYfbUh]cbU`Y`; Ya Y]bgW\UZhž`XUfi bhYf`Ui W\`^2 ghYffY]W\\g 5i »Yba]b]ghYf]i a ž`ZcfXYfh'j YfghÀf_hYb`8fi W_`Ui Z`Fi gg`UbX"`8Yfi _fU]b]gW\Y`5i »Yba]b]ghYf`5bXf]^GmV]\U`gdf]W\\h'j cb`a Ugg]j Yb 5b[f]ZZYb`i bX`ZcfXYfh'Y]bY`VYX]b[i b[g`cgY`K UZZYbfi \Y"`GY]h 6Y[]bb`XYg`?f]Y[Yg`g]bX`VYfY]hg`ØVYf`%\$\$\$`N]j]`]ghYb`[YhÒhYh k cfXYbž`k Ug`X]Y`\i a Ub]hÀfY`?f]gY`]b`XYf`I _fU]bY`k Y]hYf j YfgW\\AfZh"

$A = \frac{1}{h} \hat{A} =$

D`XYf`I _fU]bY`VYf]WX hYb`k Ygh`]WXY`A]`]hÀfj YfhfYhYf`j cb

Y`ÀbXY[Yk]bbYb`Xi fWX`X]Y`i _fU]b]gWXYb`GhfY]h_fÀZhYž`X]Y

fi gg]gWXY`Hfi ddYb`]a `CghYb`j cb`?]Yk `ni fØW_XfÀb[Yb`_cbbhYb"

8]Y`?cbhfc``Y`ØVYf`X]Y`GhÀXhY`A U_Uf]k `i bX`bU\Yni `[Ubn`=fd]b
_cbbhY`ni fØW_YfcVYfh`k YfXYb"`5``YfX]b[g'a Y`XYhY`X]Y`fi gg]gWXY

 $\label{eq:sphabx} 5fa YY\check{z}X]Y`j c``gh\grave{a}bX][Y?cbhfc``Y`@VYfX]Y`GhUXh`&g'i a `Yf`Ub[h ni `\UVYb\check{z}'k Ug'j cb'i _fU]b]gWXYf`GY]hY`VYghf]hhYb'k]fX"`8]Y`K Y`h VYf]WXhYhžXUgg`hgWXYhgWXYb]gWXY'? \grave{a}a dZYf`XUg`FUh\Ui g'j cb A Uf]i dc``@VYfbca a Yb`\UVYb'i bX`Y]b'?`]a U`XYf`5b[gh j YfVfY]hYb"$

 $\pm g[\ YgUa\ h`\ Uh'X]Y'A\]`]hAfU_h]cb'Y]bYb'bYi\ Yb'<O\Ydi\ b_h'YffY]Whzi i bX'X]Y'NU\``XYf'@i ZhUb[f]ZZY']gh'g][b]Z]_Ubh'[Ygh]Y[Ybz'a]h'a Y\fU`g'() $$ 'A YbgW\Ybz'X]Y'Ui g'VY`U[YfhYb'GhAXhYb'Yj U_i]Yfh k i fXYbz'XUfi bhYf'ZUgh'' $$$ 'Ui g'A Uf]i dc`"'8]YgY'9bhk]W_`i b[YbkYfZYb'Xi b_`Y'GW\UhhYb'Ui Z'X]Y'Ni _i bZh'XYf'I _fU]bY'i bX'XYfYb 6Yj O`_Yfi b[z'k A\fYbX'XYf'8fi W_'Ui Z'X]Y']bhYfbUh]cbU`Y; Ya Y]bgW\UZh'k AW\ghz'YbhgW\Y]XYbXY'A U\b\bU\a Yb'ni 'Yf[fY]ZYbz'i a 'XUg'k Y]hYfY'@Y]XYb'ni 'j Yf\]bXYfb"$

8 Y hU]`g	
J c f ZU``	@ ZhUb[f]ZZ
Cfh	?]Yk ž'l _fU]bY
EiY``Yb	• Vbb"XY
	k k k "X]YdfYggY"Wca
	• k k k "k Y`h"XY

6Ygi WXYb 'G]Y 'i bg 'Ui Z. 'b!U["bYh